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ENG/20M

CSCE 532 Homework 3

Problem 2.2b: Part (a): Use the languages and together with Example 2.36 to show that the class of context-free languages is not closed under intersection. Use part (a) and De Morgan’s Law (Theorem 0.20) to show that the class of context-free languages is not closed under complementation.

We proved part (a) as follows: “The grammar in our solution to Sipser Exercise 2.9 generates if we change the start variable to , and it generates if we change the start variable to . Both and are therefore CFLs. Suppose CFLs are closed under intersection. Then is a CFL. However, this contradicts Sipser example 2.36. Therefore, our assumption is incorrect, so CFLs are not closed under intersection.”

Now assume the class of CFLs is closed under complementation. We know and are both CFLs, so, and are also CFLs. Because we know that CFLs are closed under union, this implies is a CFL, and thus is also a CFL. De Morgan’s laws tell us that , and we showed above that is not a CFL. Thus, is not a CFL. This contradicts our claim that is a CFL. Therefore, our assumption is incorrect, so the class of CFLs is not closed under complementation.

Problem 2.4c: Give a context-free grammar that generates the language over the alphabet .

This is a simple one. Here’s a CFG that generates :

Clearly, we can repeatedly generate any even-length string; we only terminate when we add one additional character, which of course gives the generated string an odd length.

Problem 2.6b: Give a context-free grammar that generates the complement of the language .

This CFG generates all strings of some number of *a*s followed by the same number of *b*s. The complement of , then, is the language . If , then or . This implies that we’ll need to account for one of two possible cases. We can thus generate with the following CFG: